

THE PROBLEM WITH PEAKING IN THE ATMOSPHERIC MAGNETOHYDRODYNAMICS. LIMITING CASES

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For the mathematical modelling of highly nonequilibrium and nonlinear processes in the atmosphere based on the equations of momentum and charge transfer, a thermodynamic approach is used with the model function of sources and sinks, which is characteristic for problems with peaking, where the maximum of the velocity distribution and charge distribution in space can increase without bound for a limited time. It allows to consider the general case, taking into account the interaction between the components of the velocity vector and the electromagnetic field in the presence of sources and sinks of momentum in a flat layer. As a limiting case, we consider the transfer of momentum when its source in a nonlinear medium leads to the regime with peaking, and the development of the regime generated by a nonlinear medium itself leads to self-organization. The competition between the process increment and the propagation of momentum and charge can result in appearance of new medium characteristics, such as the spatial diameter of tornado (lightning core), in which these processes balance each other. Another limiting case is the process of charge transfer in an atmosphere considered. As a result, a more general problem may be formulated, and a joint system of equations, which not only describes the behaviour of the velocity vector for an incompressible medium in the form of parabolic equation of momentum, but also takes into account the influence of electromagnetic field, may be derived.

Introduction. The main purpose of this work is to develop a model, which would describe the phenomena that are relatively poorly understood because of strong nonlinear nonequilibrium processes taking place when these phenomena occur. As modelling objects, such different phenomena as tornado and lightning (a type of atmospheric gas discharge) have been considered. The strong nonlinearity of the processes, leading to the appearance and existence of such atmospheric phenomena, just explains the occurrence of large differences in velocity and pressure in tornado and in currents with the complex structure in the lightning. Therefore, on the way to a more common model, simple cases of momentum and charge transfer, which are the limit for the common model, were considered.

1. Transfer of momentum. Tornado. Tornado (Spanish Tornado “Whirlwind”), twister is a rapidly rotating atmospheric vortex, which occurs in cumulonimbus (thunderstorm) cloud and spreads down to the ground in the form of cloud arms or trunk of tens or hundreds of meters.

From a purely dynamical point of view, tornadoes occur due to the strengthening of existing localized vortices by an external flow of momentum [4]. Supercell thunderstorm models [1, 3, 6, 11, 13, 16] considered in the last forty years have many similarities in their basic structure. These models are convective and based on the model of mesocyclone, whose formation has been just recognized as the most useful fact to supercell identification [4]. At the same time, they differ in details from the occurring processes and, as a consequence, in initial and boundary conditions that are needed for the thunderstorm supercell to occur. Numerical models of supercell thunderstorms [8, 18] are based on the dominant mesocyclone.

Observations of tornadoes have a rich history. Brooks was the first observer, who put forward the generally accepted assumption that the funnel is a part of the parent cloud, the structure and dynamics of which represent a small tropical storm with a helical structure [2]. Numerous observations of the parent cloud showed the presence of long vortices in the horizontal plane in them; the vertical poles (funnels), whose walls have extremely fast spinning [7], are their continuation [17]. In his monograph, Flora [5] notes that "the distinction between the strongest winds in the body of the funnel and the stationary air at its periphery is so sharp that it causes a number of damaging effects." Observations of the actual tornado, therefore, indicate a strong nonlinearity and nonequilibrium of the processes during the formation and existence of a tornado that does not allow to create a perfect model of this exotic phenomenon.

For the mathematical modelling of highly nonequilibrium and nonlinear processes in tornado, the authors propose an approach based on the nonlinear equations of momentum transfer with the model source and sink function. This approach can be assigned to the problems with peaking considered by Samarskii [9, 14, 15]. But in contrast to the well-known problems with peaking Samarskii applied to the thermal fields during combustion processes, similar problems for the transfer of momentum (and charge, see section 2) in the atmosphere has not been previously formulated and solved. For the first time in the model development, a thermodynamic description not considered previously has been used to identify new principles of self-organization in the atmosphere.

Basing on the equations of momentum with a nonlinear function of sources and sinks, a nonlinear hydrodynamic model of strongly nonequilibrium processes in the atmosphere during an intense vortex formation was proposed by the authors. A thin layer of unit volume, parallel to the surface of the earth, is allocated in the tornado. In this layer, there are sources of movement and its sinks depending on the horizontal velocity vector and its modulus. The temperature and other air characteristics change only with the altitude. A "layered" model is used to simplify the modelling of vortices existing in a three-dimensional (3D) region of space as well as to facilitate numerical calculations.

We used the law of energy conservation [21] per unit volume of the air layer at a certain height with momentum sources and sinks. After differentiation with respect to time as well as from the principle of spatial locality, we derive the equation of momentum with the function of sources and sinks similar to the Samarskii problem with peaking [15]:

$$\frac{\partial \vartheta}{\partial t} = \hat{\nu} \Delta \vartheta + \frac{T_0}{\rho} \left(\frac{\partial \sigma^e}{\partial \vartheta} \right), \quad \frac{T_0}{\rho} \left(\frac{\partial \sigma^e}{\partial \vartheta} \right) = q \vartheta - \hat{\alpha} |\vartheta|^2 \vartheta. \quad (1)$$

For a two-dimensional (2D) case, the velocity vector consists of two components, so to account for the cross effects (anisotropy of the medium), the cinematic viscosity $\hat{\nu}$ and $\hat{\alpha}$ (a parameter characterizing nonlinear sinks) are introduced as second-rank tensors:

$$\hat{\nu} = \begin{pmatrix} \nu_1 & -\nu_2 \\ \nu_2 & \nu_1 \end{pmatrix}, \quad \hat{\alpha} = \begin{pmatrix} \alpha_1 & -\alpha_2 \\ \alpha_2 & \alpha_1 \end{pmatrix}. \quad (2)$$

Here T_0 is the layer temperature, ν_1 is its own kinematic viscosity ($[\nu_i] = \text{m}^2/\text{s}$), ν_2 is a viscosity related to the symmetric cross interaction between the velocity components in the layer, q is a constant characterizing the sources of momentum ($[q] = 1/\text{s}$), $\hat{\alpha}$ is a tensor characterizing nonlinear sinks ($[\alpha_i] = \text{s}/\text{m}^2$).

In this case, the Onsager coefficients $L_{ik} = T_0 \rho \nu_{ik}$ ($[L_{ik}] = \text{Pa} \cdot \text{sK}$), considering the anisotropy of the medium and corresponding to the antisymmetric

Casimir reciprocity relations $L_{ik} = -L_{ki}$ for generalized thermodynamic forces, are odd with respect to time (β -type) when considering the fields of vector quantities (magnetic field, velocity fields, etc.). Following Gyarmati ideas, we assume them as axioms, which are experimentally confirmed for vector fields.

In this case, all of the bulk forces

$$\mathbf{f} = \nabla p / \rho + (\vartheta \nabla) \vartheta + q \vartheta - \hat{\alpha} |\vartheta|^2 \vartheta$$

contain the pressure gradient and are functions of the motion sources and sinks. Thus, the nonpotentiality of the flow is related just to the nonlinearity of the medium, i.e. in the presence of the source and sink function. This can lead, in some cases, to a regime with peaking, i.e. sharp increases in velocity amplitudes during some short time. The development of this mode leads to a self-organization. In this formulation, the formation of dissipative structures localized in space is possible. In these structures, the speed may increase limitedly (or unlimitedly).

This approach allows one to derive a system of equations for the velocity components in the case of an incompressible fluid in the dimensionless form as a 2D Kuramoto-Tsuzuki equation [10] for the atmospheric layer:

$$\frac{\partial \Phi}{\partial t} = \nu_1^* (1 + ic_1) \left(\frac{\partial^2 \Phi}{\partial x^{*2}} + \frac{\partial^2 \Phi}{\partial y^{*2}} \right) + q^* \Phi - \alpha_1^* (1 + ic_2) |\Phi|^2 \Phi, \quad (3)$$

where $\Phi = \vartheta_x^* + i\vartheta_y^*$ (a complex value of the flow velocity in the atmospheric layer); $c_1 = \nu_2^*/\nu_1^*$ is related to viscosity, $c_2 = \alpha_2^*/\alpha_1^*$ is due to sinks. The superscript '*' denotes the dimensionless form of the parameters on certain scales identified at observing a tornado.

In the nonequilibrium thermodynamics, it is assumed to characterize the processes within the system under the influence of external environment by the so-called entropy production σ^i per unit volume of the layer

$$\begin{aligned} \sigma^i = & \frac{\rho \nu_1}{T_0} \left[\left(\frac{\partial \vartheta_y}{\partial x} \right)^2 + \left(\frac{\partial \vartheta_x}{\partial y} \right)^2 \right] + \\ & \frac{\rho \nu_2}{T_0} \left[\left(\frac{\partial \vartheta_x}{\partial x} \right)^2 + \left(\frac{\partial \vartheta_y}{\partial y} \right)^2 \right] - \\ & \frac{\rho (\nu_1 + \nu_2)}{T_0} \left[\frac{\partial \vartheta_x}{\partial x} \frac{\partial \vartheta_y}{\partial x} - \frac{\partial \vartheta_x}{\partial y} \frac{\partial \vartheta_y}{\partial y} \right]. \end{aligned} \quad (4)$$

There are also other local thermodynamic characteristics, such as the external flow of entropy σ^e

$$\sigma^e = \frac{q\rho}{2T_0} \vartheta^2 - \frac{\alpha_1 \rho}{4T_0} \vartheta^4 - \frac{\alpha_2 \rho}{3T_0} \vartheta_x \vartheta_y \vartheta^2. \quad (5)$$

and the rate of change of entropy dS/dt equal to their sum. It is believed that in the self-organizing systems the full change of entropy decreases with time: $dS/dt < 0$.

The boundary conditions were taken based on the simple assumption of lack of air flow at the cell boundary. As initial conditions, some small initial spin of the air stream at the center was set.

The results of numerical simulations give a good resemblance to the observed physical phenomena. The unusual behaviour of the velocity near the center of the area is registered. We can assume that the emergence of a stationary core in a flat tornado layer (the structure is localized in space) is the result of peaking (flow non-potentiality). Large amplitudes of speeds of the tornado funnel rotation remain

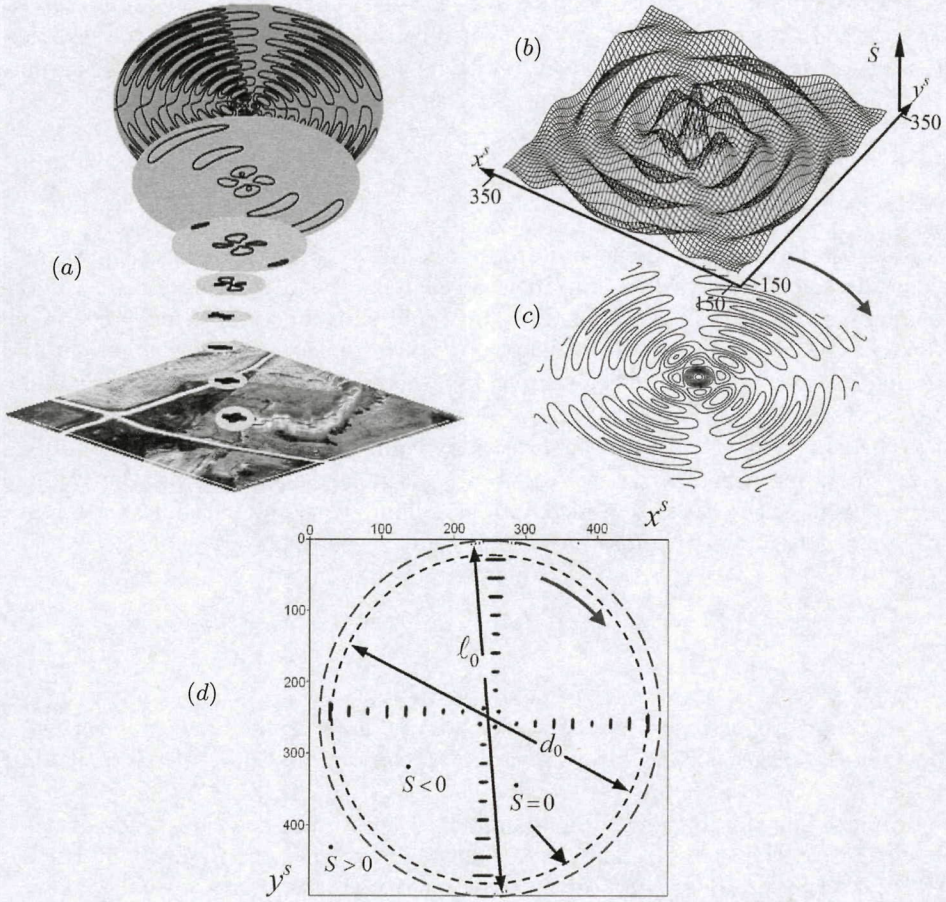


Fig. 1. The entropy production in the tornado basin from the ground up to 3.5 km (a) and the total entropy change at $h = 0$ (b), its projection (c) characterizing the region of self-organization (d) and the boundary of the localized vortex structures.

constant with a relatively small number of spiral wave turns. The velocity amplitudes decrease with the increasing modulus of the spiral wave topological charge. During the formation of additional branches, maximum and average velocities are reduced and may take zero value.

Thus, this approach allows us to explain the nonlinear mechanisms of layered momentum transfer in the atmosphere and also the origin, evolution and decay of large, medium and small atmospheric vortices as dissipative thermodynamic structures. Competition of the pulse increment and propagation processes in a viscous medium leads to the appearance of a linear size l_0 – the spatial diameter of a self-organized structure, at the border of which the rate of entropy variation changes its sign. One can set a dependence of the initial conditions on humidity, which varies with the altitude.

Humidity, which determines the radial component of the emerging spiral wave, at its reduction in the lower layers leads to a significant depletion of the vortex structure around the trunk and even to disappearance, and, hence, to a more realistic result when the trunk is only visible between the earth and the cloud which forms the tornado (Fig. 1a). As it also follows from Fig. 1, the multivortex structure is typical. There are mesovortex areas pulled to the periphery, i.e. ordered

Table 1. Spatial characteristics of tornado with the diameter pool $l_0 = 500$ m, $|m| = 1$, (comparative table).

Flow type	NP	NP	P
Characteristics	ϑ^*	∇p^*	∇p^*
Self-organization zone diameter d , [m]	439	459	426
Inner core diameter, [m]	11	9	9
Outer core diameter, [m]	20	20	21
Width of the pressure equalization ring to atmospheric pressure, [m]	30	20	37

dissipative structures, which are a nonlinear system response to the external influences in the form of external entropy flow. As for the medium-sized vortices, the characteristics of the mesovortex zones are closed streamlines. At the center of these structures, there are zones of self-organization, where entropy decreases with time (Fig. 1d). At the tornado core, the entropy change rate takes the highest negative values (the four black dots in the center of Fig. 1d), which confirm the formation of the most stable and strongest vortices exactly near the tornado core.

Table 1 presents the spatial characteristics of a comparatively small-sized tornado defined by numerical methods for two versions of the turbulent flow approximation: nonpotential (NP) and potential (P). The nonpotential flow means that the function of the pressure gradient is related to the velocity in terms of the sources and sinks. Determination was made on the velocity (ϑ^*) and pressure (∇p^*) gradients. The obtained characteristics are in good agreement with the observations of real tornado, that is, according to the authors, proves the conformity between theory and experiment.

Thus, the thermodynamic model is fundamentally different from the convective models and makes it possible to construct a model, which would reflect some of the principal features of the real observed tornado. These are such features as the presence of an almost perfectly circular in cross-section trunk, the existence of significant gradients of velocity and pressure at the edge of the tornado basin and in the area of the vertical vortex tube [5], along with the complete absence of rotation and discharge in the tornado center [7]. The considered approach allows us to formulate conditions of self-organization in tornadoes currently determined by the authors. This approach allows to expand the problem in asymptotics to the conditions of formation of cyclones in the atmosphere. Also, some physical restrictions on the horizontal velocity, the intensity of sources and sinks and the stability condition of dissipative structures are specified. These restrictions cannot be obtained explicitly using the convective and other models.

Such nonlinear hydrodynamic model based on a thermodynamic approach can be used to describe cyclones, tornadoes, and other exotic natural phenomena.

1.1. Condition for the stability of vortex flows in a tornado. Stability analysis of solutions of the Kuramoto–Tsuzuki equation [10] yields the following result:

$$(c_1^2 + 1)k^4 + 2(1 + c_1c_2)k^2 > 0, \quad k = \pi/l_0. \quad (6)$$

In order to satisfy the inequality for any value of k , it is necessary to limit the constants c_1, c_2 : $-1 < c_1c_2 < 1$. This condition is a criterion for the stability of turbulent structures in atmospheric vortices. Therefore, it is a condition of release of such vortices in the steady state.

The emergence of a stationary core in a plane layer of tornado – space localized figure – is the result of sharpening and confirmed in [12], which describes the unusual properties of the presence of large pressure gradients and velocities in tornadoes.

1.2. Thermodynamic conditions of self-organization. The sum of external (σ^e) and internal (σ^i) thermodynamic fluxes is characterized by the full rate of entropy change \dot{S} . The decrease in entropy with time corresponds to the processes of self-organization of vortex structures. The transition from entropy reduction to its growth makes it possible to define the boundary of the vortex tornado basin, in which stable self-organized vortex systems exist. As follows from Fig. 1, the entropy of each tornado vortex layer decreases with time: $\dot{S} < 0$. This is achieved with a value of sources of momentum, i.e. the reversible flows of entropy $\sigma^e < 0$, which are compared with the entropy production $|\sigma^e| > \sigma^i$. The condition of the limited lateral tornado is a value $\dot{S} = 0$, that is $\sigma^e + \sigma^i = 0$ at the vortex-formation boundary (Fig. 1d). Outside the vortex tornado basin, entropy increases ($\dot{S} > 0$). The zone of self-organization decreases with time, which finally leads to tornado collapse.

1.3. Hydrodynamic conditions of self-organization. This conditions in a tornado, established for the first time, follow from the thermodynamic conditions. The boundedness of tornado in height is owing to the absence of vortex formation due to high viscosity. The transverse size of tornado is caused by the competition between the rate of velocity and the momentum distribution in terms of the viscosity. Positive feedback (FB) between the projections of horizontal velocities leads to sharpening. The intensity of the sources and sinks of movement causes the limitation on the modulus of the horizontal velocity:

$$v^{*2} \geq \frac{4q^*}{\alpha_1^*(1 - 4c_2^2/9)}.$$

Numerical calculations of such problem with the sharpening have shown the possible existence of enormous velocities of air motion directed to the nucleus. Also, this approach makes it possible to calculate the spatial distribution of pressure in the tornado basin with a change in height if compared with the external pressure.

Restriction on the constant of the Kuramoto–Tsuzuki equation for the emergence of self-organization is the condition: $c_2^2 \leq \alpha_2^{*2}/\alpha_1^{*2}$. The left twisting condition in the topological charge is: $m = -1$, right – $m = +1$. The restriction on the modulus of the topological charge for the formation of a tornado is $|m| \leq 2$. The transition to the values of the topological charge $|m| > 2$, firstly, leads to a huge cyclone model, i.e. atmospheric structures without a trunk with a significantly lower velocities, and, secondly, to the emergence of ‘eye’, i.e. the cyclone core.

2. Charge transfer. Lightning. The linear lightnings usually observed in the atmosphere are the so-called electrodeless discharges, because they begin (and terminate) in clusters of charged particles. This defines their some still unexplained properties that distinguish the lightning from discharges between electrodes [19]. Thus, the lightning can be at least a few hundred meters and it occurs in an electric field much weaker than the field during the interelectrode discharges. Collection of charges carried by the lightning occurs in milliseconds from billions of small particles well-insulated from each other in the bulk of several cubic kilometers. The lightning development in thunderclouds is most studied, and the lightning can take place in the clouds themselves, i.e. intracloud lightning, and can strike the ground, i.e. ground lightning.

As another limiting case, the authors consider the charge transfer in the atmosphere, where it is a competition between the increment of the electric potential

and the charge propagation over a certain spatial diameter, on which these processes balance each other.

Likewise the problem of momentum transfer from the law of energy conservation per unit volume of the air layer at a certain height with the sources and sinks of charge, after differentiation with respect to time, as well as from the principle of spatial locality, we derive a local parabolic conductivity equation with the function of sources and sinks similar to the Samarskii problem with peaking [15]:

$$\frac{d\varphi}{dt} = \frac{\gamma}{C}\Delta\varphi + \frac{T_0}{C} \left(\frac{\partial\sigma^e}{\partial\varphi} \right), \quad (7)$$

where C is the capacitance per unit volume ($[C] = \text{F/m}^3$), γ is the conductivity of the medium ($[\gamma] = \text{m}^{-1}\Omega^{-1}$), q is a constant characterizing the intensity of the sources ($[q] = 1/\text{s}$), α is the intensity of nonlinear sinks ($[\alpha] = \text{s}^{-1}\text{V}^{-2}$). Eq. (7), in the case $d\varphi/dt = 0$, gives the following expression for the external charges as a function of coordinates and time: $\rho(x, t) = -T_0\epsilon_0 (\partial\sigma^e/\partial(\gamma\varphi))$, where ϵ_0 is the constant permittivity.

The conductivity Eq. (7) completely replaces the system of Maxwell equations in the absence of magnetic charges ($\text{div}\mathbf{B} = 0$, $d\varphi/dt \neq 0$) starting with the equations

$$\frac{\partial\mathbf{B}}{\partial t} = \frac{1}{\mu\mu_0\gamma}\Delta\mathbf{B} + \text{rot}[\vartheta\mathbf{B}], \quad \frac{\partial\mathbf{E}}{\partial t} = \frac{c^2}{\epsilon\mu}\text{rot}\mathbf{B} - \frac{\mu_0 c^2}{\epsilon}\mathbf{J}_j, \quad (8)$$

where $\mu_0 = 4 \cdot 10^7 Hn$ is the constant magnetic permeability, ϵ and μ are the permittivity and the magnetic permeability of the medium. Applying the operation of rotor to the first equation, after subtracting from it the second equation with the operation of gradient, we also get Eq. (7).

2.1. The Kuramoto–Tsuzuki equation for the lightning. Let us introduce a description of the considered area of a thin layer by considering two potentials $\varphi = \varphi_1 + i\varphi_2$ and tensors for the self electrical conductivity $\hat{\gamma}'$ and for the constants of sources q and sinks $\hat{\alpha}$. Separation of the potential into the real and imaginary parts indicates that there are two processes: 1) the charge transfer in the lightning described by the real part of the potential φ_1 , and 2) the skin effect described by its imaginary part φ_2 . As a result, by analogy with the model of momentum, we obtain a 2D system of two nonlinear differential equations of parabolic type with the function of sources and sinks, which, in the case of the considered generalized potential, can be written in the reduced form:

$$\frac{\partial\varphi^*}{\partial t} = \gamma_1^* (1 + ic_{\varphi_1}) \left(\frac{\partial^2\varphi^*}{\partial x^{*2}} + \frac{\partial^2\varphi^*}{\partial y^{*2}} \right) + q_{\varphi}^* \varphi^* - \alpha_{\varphi_1}^* (1 + ic_{\varphi_2}) \varphi^* |\varphi^*|^2. \quad (9)$$

Here γ_1^* is the self electrical conductivity, which is responsible for the charge transfer and for the skin effect in the lightning cord, $c_{\varphi_1} = \gamma_2^*/\gamma_1^*$ and γ_2^* are the cross ratios responsible for the interaction of the skin layer and the main conduction current. The nonlinear terms depend on the parameters q_{φ}^* , $\alpha_{\varphi_1}^*$, $c_{\varphi_2} = \alpha_{\varphi_2}^*/\alpha_{\varphi_1}^*$ and describe the complex behaviour of the components for any selected air layer at a certain altitude. The presence of the positive constant q in the right side of Eq. (9) indicates the presence of positive FB and can lead to a sharp increase in both components of the generalized electric potential. Terms with the negative q are responsible for the presence of strong nonlinear negative FB and can reverse the effect of the positive FB under the strong increase in amplitude of φ_1 , φ_2 . Instability conditions for the solutions, which lead to the destruction of the lightning cord, can be obtained by analogy with the previous section as expressions for the

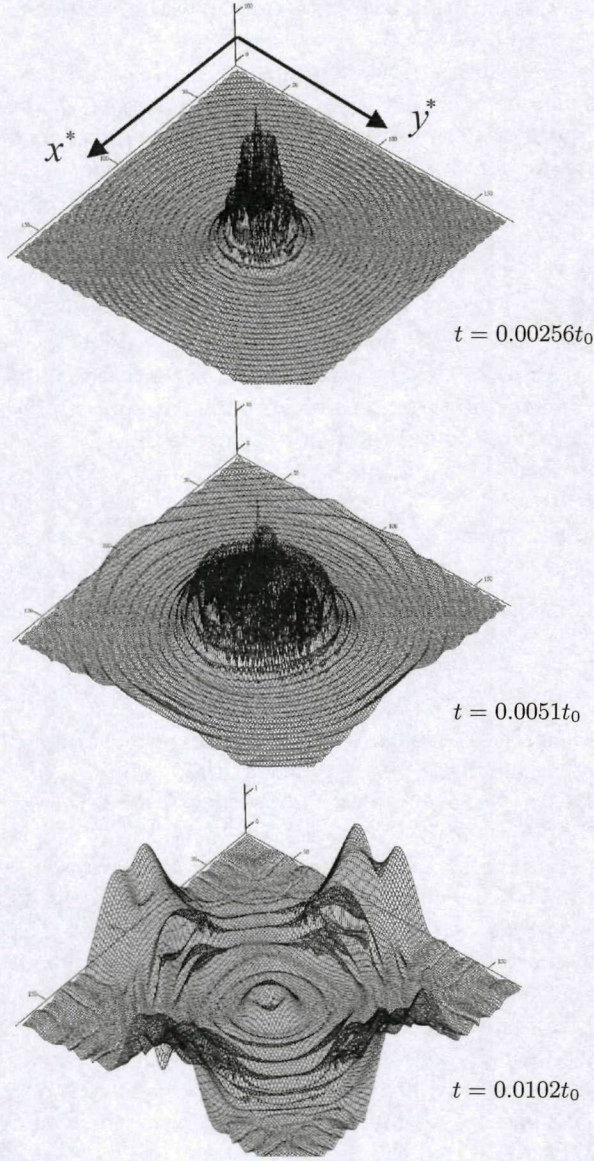


Fig. 2. Peaking mode of atmospheric discharge. Parameters, for which Eq. 9 has been solved: $c_1 = 2$, $c_2 = -1$, $q^* = 1$, $\alpha_1^* = 1$, $\alpha_2^* = -1$. Calculated in 200×200 pixels with increments of x and y : $\Delta_x = \Delta_y = 0.00628$.

thermodynamic characteristics: the production (σ^i) and the external flow (σ^e) of entropy, and the full change rate (dS/dt). The initial and boundary conditions to compare the results were taken as in the first section, but the parameters of Eq. (9) were chosen to obtain unstable solutions.

Numerical calculations of Eq. (9) have shown that there is a super-fast increase and subsequent redistribution of the electric current (Fig. 2) presented in a plane perpendicular to the lightning cord. The lightning cord cross-section has a fractal structure.

This is due to the fact that the positive feedback leads to the accelerated increase of the generalized potential over the entire space and also provides super-fast increase of the electric current.

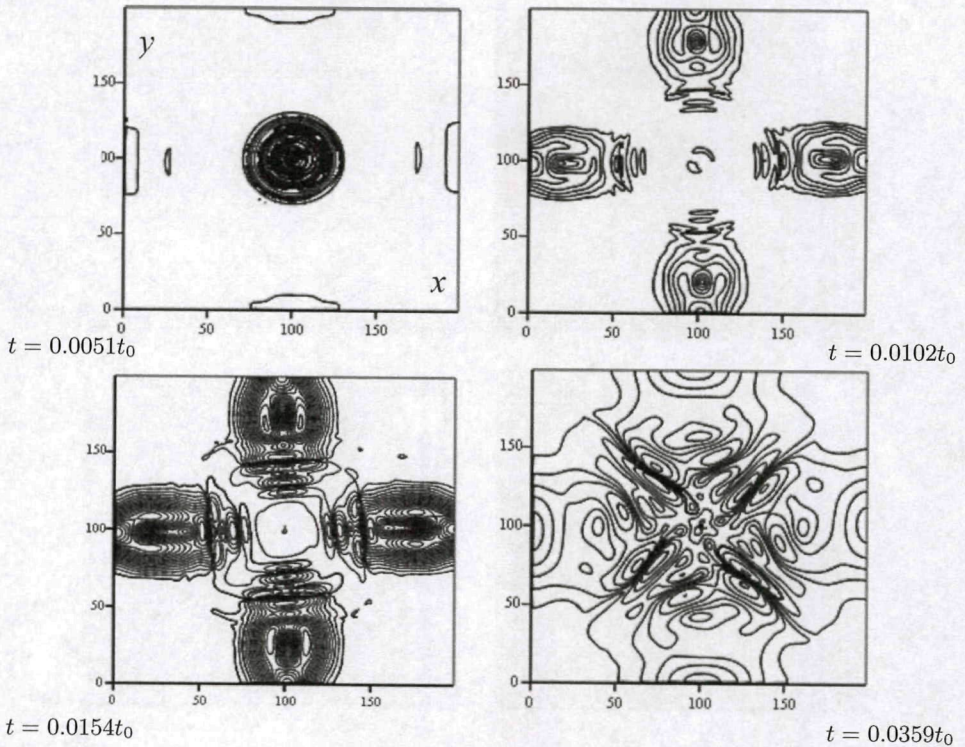


Fig. 3. Cross-section profile of the dimensionless total electric current for the lightning cord.

2.2. *The interaction of skin effect and conductivity in the medium.* The condition of solution instability is a criterion for the lightning cord destruction and subsequent occurrence of side branches from it. Also, the total electric flux was determined. It can be represented as a complex vector: $\mathbf{J}^* = \mathbf{J}_{\text{Re}}^* + i\mathbf{J}_{\text{Im}}^*$.

The transition to the cross ratios γ_2 high values (which increase the constant $c_{\varphi 1} = \gamma_2^*/\gamma_1^*$ under the instability conditions) leads to the destruction of the lightning cord subsequently. When the diameter of the lightning cord is $d^* = d = 0.3$ m, the maximum observable area is $x_0 = 1.25$ m.

The analysis of the numerical solutions for the module of the total current $\mathbf{J}^* = \mathbf{J}t_0/(Cl_0\varphi_c)$ (Fig. 3) and density of external charges $\rho^* = \rho\gamma t_0/(\epsilon_0 l_0\varphi_c)$ (Fig. 4) has allowed to detect the fractal structure not only for the lightning cord, but for side branches, which arise after the destruction of the cord after a certain time. The destruction of the cord is enhanced not only due to the redistribution of the external charges in the lateral regions (Figs. 3,4), but also due to the appearance of bias current. Thus, the nature of the bias current is associated with the presence of external entropy flows across the boundary of a layer considered. Therefore, the bias currents determine the sign of σ^{e*} , which defines the possibility of emergence of self-organized structures. After a certain time, the side branches disappear, forming a complex symmetric structure, which then fades, and reappear as in the current as in the external charges.

2.3. *The transition from stability to instability in the lightning cord.* Based on numerical calculations, the dependence of the diameter of the lightning cord on the constant $c_{\varphi 1}$ in Eq. (9) has been investigated. The transition from stability in the lightning cord to unstable processes, under which the cord collapses, has been

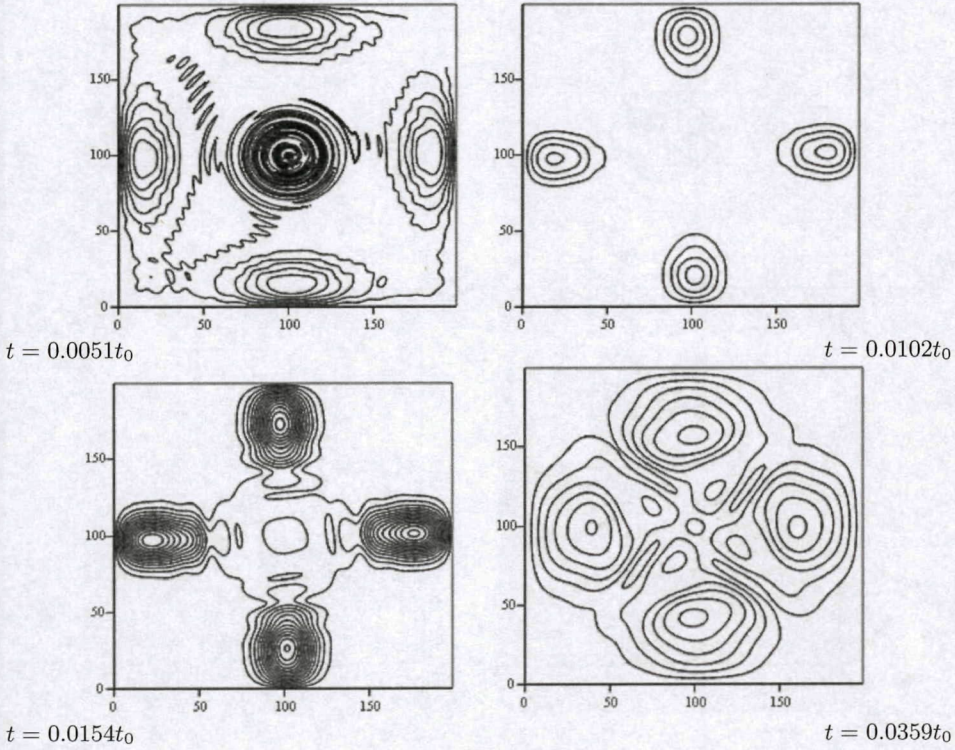


Fig. 4. Cross-section profile of the dimensionless density of external charges for the lightning cord.

calculated. Non-analytic behaviour (break) at the critical value $c_1 = 1$ is similar to the behaviour of the thermodynamic functions at phase transition. This is a consequence of the above-described strong interaction of the skin effect and the main conduction current. As shown by numerical calculations, in the field I , the bias current $\mathbf{J}_e = \text{rot}\mathbf{H}$ is small. After the function breaks, these currents become significant and lead to the appearance of a self-organized zone.

2.4. Thermodynamic characteristics of the discharge. The electric flux density and the density of external charges do not give the spatial and temporal distribution of energy in the lightning and in the dissipation field. To identify areas of self-organization in the lightning, thermodynamic characteristics were calculated separately. It was found that in the the initial time external flows σ^{e*} through the cord boundary were maximum, which may indicate the nucleation an external charge transfer process to the side branches. The entropy change rate dS^*/dt as well as the entropy production σ^{i*} is similar in structure and at the initial time only occupies the central region, which may be associated with the formation of the lightning cord in this area. The bias current is responsible for the circulation of the magnetic field and is directed opposite to the conduction current, and the difference between them leads to the destruction of the cord.

Conclusion. A model of momentum transfer in the atmosphere has been developed. The conditions of stability and self-organization are defined on the basis of the model. This model describes the phenomenon of tornado. Thermodynamic and hydrodynamic conditions of the self-organization for this model are determined.

A model of charge transfer in the atmosphere has been developed. It describes the formation of side branches from the lightning cord. Unstable solutions, leading to the decay of the main cord to form the side branches generated by its fractal structure, are considered. Although the problem of gas discharge (lightning) belongs to problems with instability, the self-organization conditions obtained allow to determine the conditions for the stability of gas discharge, which are of great importance in solving some technical problems [20].

A generalized model of joint momentum and charge transfer based on the limiting cases has been also considered. The boundary-value problem has been formulated.

For all models, the thermodynamics has been developed, and the expressions for the entropy production rate, external flow rate and for the resulting expressions for the entropy and free energy change rates are defined.

As a result of the thermodynamic approach for the generalized model, the conditions of self-organization can also be obtained.

Basing on the limiting cases of momentum and charge transfer, a generalized model of joint transfer of momentum and charge can be developed. In this case, one must consider the right-hand side of the equation of momentum (Eq. (3), where the term $(\chi/\rho) \operatorname{div} (\nabla \varphi)$ is responsible for the contribution of the electromagnetic field, and add this system of equations to the Maxwell equations and constitutive equations. As a result, a consistency of the resulting system of equations with both the Navier–Stokes equations and the Maxwell equations may be shown, and the boundary-value problem may be formulated, for example, to describe nonpotential turbulent flows in plasma. But when applying this generalized model, there arise many difficulties, such as cross electrohydrodynamic effects, parameters from experiments required for the model, and comparison of the results of simulation with experiment. Discussion of these problems goes far beyond the scope of this work and therefore is not given.

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